Commonly used DSP formulae

IIR filter pre-warp

$$\omega_a = \frac{2}{\Delta t} \tan\left(\frac{\omega_d \ \Delta t}{2} \right) \tag{5.16}$$

Bilinear transformation

$$s = \frac{2(1 - z^{-1})}{\Delta t(1 + z^{-1})} \tag{5.19}$$

For a BPF replace z^{-1} in the LPF by:

$$z_{LP}^{-1} = \frac{-z^{-1}(z^{-1} - \alpha)}{(1 - \alpha \ z^{-1})}$$

where:

$$\alpha = \frac{\cos\left[\dfrac{\pi(\omega_{cu} + \omega_{cl})}{\omega_s} \right]}{\cos\left[\dfrac{\pi(\omega_{cu} - \omega_{cl})}{\omega_s} \right]}$$

The LPF cutoff frequency $\omega_{C_{LP}} = \omega_{cu} - \omega_{cl}$ and the stopband frequency $\omega_{S_{LP}} = \omega_{C_{LP}} + \Delta\omega$, where $\Delta\omega$ is the transition bandwidth. This yields:

$$z = z_{BP} = \frac{1}{2} \left[\alpha(z_{LP} + 1) \pm \sqrt{\alpha^2 \ (z_{LP} + 1) - 4 \ z_{LP}} \right]$$

$$p_{BP} = \frac{1}{2} \left[- \alpha(p_{LP} + 1) \pm \sqrt{\alpha^2 \ (p_{LP} + 1) - 4 \ p_{LP}} \right]$$

For a BSF replace z^{-1} in the LPF by: $+z^{-1}(z^{-1} - \alpha) / (1 - \alpha \ z^{-1})$. This time the LPF cutoff should be $\omega_{C_{LP}} = (\omega_s/2) - (\omega_{cu} - \omega_{cl})$. This yields:

$$z_{BS} = \frac{1}{2} \left[- \alpha(z_{LP} - 1) \pm \sqrt{\alpha^2(z_{LP} - 1)^2 + 4 \ z_{LP}} \right]$$

$$p_{BS} = \frac{1}{2} \left[- \alpha(p_{LP} - 1) \pm \sqrt{\alpha^2(p_{LP} - 1)^2 + 4 \ p_{LP}} \right]$$

FIR Filter Coefficients

$$c_n = \frac{\Delta t}{\pi} \int_0^{\pi/\Delta t} H_D(\omega) \cos(n\omega\Delta t) \ d\omega \tag{6.6}$$

Window Characteristics - Table 6.1

Window	Transition Band (Hz)	Stopband Rejection (dB)
rectangular	$\frac{1}{N\Delta t}$	21
Hanning	$\frac{3.1}{N\Delta t}$	44
Hamming	$\frac{3.3}{N\Delta t}$	53
Kaiser $\beta = 6$	$\frac{4}{N\Delta t}$	63
Blackman	$\frac{5.5}{N\Delta t}$	74
Kaiser $\beta = 9$	$\frac{5.7}{N\Delta t}$	90

Hanning Window

$$w_n = \frac{1}{2}\left(1 + \cos\left(\frac{n\pi}{M} \right)\right) \tag{6.10}$$

Hamming Window

$$w_n = 0.54 + 0.46 \cos\left(\frac{n\pi}{M} \right) \tag{6.11}$$

Filter $h(t)$ matched to signal $f(t)$

$$h(t) = kf^*(t_m - t) \tag{6.23}$$

or equivalently

$$H(\omega) = kF^*(\omega) \exp(-j\omega t_m)$$

z-transform of Filter Output Autocorrelation Sequence

input x output y

$$S_{yy}(z) = H(z)\, H(z^{-1})\, S_{xx}(z) \tag{7.12}$$

z-transform of Filter Input/Output Cross-Correlation Sequence

input x output y

$$S_{xy}(z) = H(z)\, S_{xx}(z) \tag{7.15}$$

The noise variance at the output of a filter $\Sigma_{i=0}^{N} a_i\, z^{-i}/(1 - \Sigma_{i=1}^{N} b_i\, z^{-1})$ is given by the solution to:

$$\mathbf{B}\, [\, c_0\; c_1\; c_2\; \cdots\; c_N\,]^T = \mathbf{a} \tag{7.19}$$

where

$$\mathbf{B} = \left\{ \begin{bmatrix} 1 & -b_1 & -b_2 & \cdots & -b_N \\ 0 & 1 & -b_1 & \cdots & -b_{N-1} \\ 0 & 0 & 1 & \cdots & -b_{N-2} \\ . & . & . & & . \\ . & . & . & \cdots & . \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & \cdots & -b_{N-2} & -b_{N-1} & -b_N \\ -b_1 & \cdots & -b_{N-1} & -b_N & 0 \\ -b_2 & \cdots & -b_N & 0 & 0 \\ . & & . & . & . \\ . & \cdots & . & . & . \\ -b_N & \cdots & 0 & 0 & 0 \end{bmatrix} \right\};$$

$$\mathbf{a} = \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_N \\ 0 & a_0 & a_1 & \cdots & a_{N-1} \\ 0 & 0 & a_0 & \cdots & a_{N-2} \\ . & . & . & & . \\ . & . & . & \cdots & . \\ 0 & 0 & 0 & 0 & a_0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ a_N \end{bmatrix}$$

The noise variance at the output is: $\sigma_y^2 = \sigma_x^2 \, 2c_0$

Wiener Filter

input y output x

$$\mathbf{\Phi}_{yy} \, \mathbf{h}_{opt} = \mathbf{\Phi}_{yx} \tag{8.8}$$

Minimum MSE

$$\eta_{opt} = \mathrm{E}[\, x^2(n) \,] - \mathbf{h}_{opt}^T \, \mathbf{\Phi}_{yx} \tag{8.10}$$

LMS Adaptive Filter

input y output x

$$e(n) = x(n) - \mathbf{h}^T(n-1) \, \mathbf{y}(n) \tag{8.29}$$

$$\mathbf{h}(n) = \mathbf{h}(n-1) + 2 \, \mu \, \mathbf{y}(n) \, e(n) \tag{8.30}$$

LS filter

$$\mathbf{R}_{yy}(n)\mathbf{h}(n) \; = \; \mathbf{r}_{yx}(n) \tag{8.13}$$

RLS Adaptive Filter

input y output x

$$\mathbf{h}(n) = \mathbf{h}(n-1) + \mathbf{R}_{yy}^{-1} \, \mathbf{y}(n) \, e(n) \tag{8.18}$$

$$\mathbf{R}_{yy}^{-1}(n) = \mathbf{R}_{yy}^{-1}(n-1) - \frac{\mathbf{R}_{yy}^{-1}(n-1) \, \mathbf{y}(n) \, \mathbf{y}^T(n) \, \mathbf{R}_{yy}^{-1}(n-1)}{\left(1 + \mathbf{y}^T(n) \, \mathbf{R}_{yy}^{-1}(n-1) \, \mathbf{y}(n) \right)} \tag{8.20}$$

DFT

$$X(k) = \sum_{n=0}^{N-1} x(n) \exp\left(-j \frac{2\pi nk}{N} \right) \tag{9.12}$$

Inverse DFT

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \exp\left(j \frac{2\pi nk}{N} \right) \tag{9.13}$$

Power Spectral Density

$$S_{xx}(\omega) = \sum_{m=-\infty}^{\infty} \phi_{xx}(m) \exp(-j\omega m\Delta t) \tag{9.24}$$

Autocorrelation

$$\phi_{xx}(m) = \frac{\Delta t}{2\pi} \int_{0}^{2\pi/\Delta t} S_{xx}(\omega) \exp(j\omega m\Delta t)\, d\omega \tag{9.25}$$

Linear Prediction

$$\hat{x}(n) = \sum_{i=1}^{P} a_i \, x(n-i)$$

Prediction Error

$$e(n) = x(n) - \hat{x}(n)$$

Order-P AR Spectral Estimate

$$\hat{S}_{xx}(\omega) = \frac{\hat{\sigma}_e^2}{\left| 1 - \sum_{n=1}^{P} a_n \exp(-jn\omega\Delta t) \right|^2} \tag{9.38}$$